Formulas (3.2) show that when the parameter α tends to zero, the limiting expressions for an almost ideal gas pass smoothly to the corresponding expressions for the ideal gas. Thus, by taking the heat conduction into account we can obtain not only a finite value for the temperature at the center of explosion, but also a correct limiting passage from the imperfect to the ideal gas case. Corrections to the solution of the problem of explosion in an ideal thermally conducting gas caused by the nonideal character of the medium can be easily obtained from the system (3.1).

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CHARACTERISTICS OF TRAJECTORIES OF A FIELD GENERATED BY RANDOMLY DISTRIBUTED SOURCES

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We investigate the statistical characteristics of trajectories of a (scalar, vector, tensor) random field σ generated by independently distributed sources. Formulas obtained for the trajectory of a component σ of such a field along an arbitrary straight line x_0, define the mean values of the following characteristics (see Fig. 1): $l^+ = l^+(\tau)$ and $l^- = l^-(\tau)$ are the distances between two neighboring upcrossings and downcrossings of the level τ; $\xi = \xi(\tau)$ is the duration of an upwards excursion across τ, while $\lambda^+ = \lambda^+(\varkappa)$ and $\lambda^- = \lambda^-(\varkappa)$ are distances separating the consecutive points on the trajectory possessing the same first order derivative with respect to $\varkappa$, with positive and negative curvature, respectively. The formulas differ from those given by the general mathematical theory of trajectories of stationary random processes; they can be applied in practice to obtain the characteristics of traject-

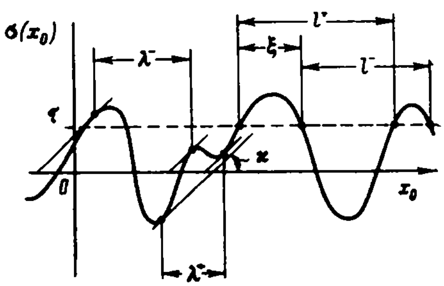

Fig. 1

ories for a wide range of physical fields.

For a random stationary process (scalar field on a straight line) with a differentiable trajectory, the mean number of upcrossings $\mu^+(\tau)$ of the level τ per unit length can be defined using the two-dimensional distribution density $\varphi(y, z, l)$ of a component of the field at two points on the observer line x_0 separated by the distance l, in the manner analogous to that employed in [1]

$$\mu^+(\tau) = [\langle l^+(\tau)\rangle]^{-1} = \lim_{l\to+0} \frac{P^+(l)}{l} \tag{1}$$

$$P^+(l) = \int_{-\infty}^{\tau} dy \int_{\tau}^{\infty} dz\,\varphi(y, z, l)$$

The mean number of downcrossings $\mu^-(\tau)$ of the level τ is defined in the same manner. The total number of crossings of the level τ per unit length is $\mu(\tau) = \mu^+(\tau) + \mu^-(\tau)$.

For the majority of real fields the two-dimensional distribution $\varphi(y, z, l)$ is unknown. If the field σ is generated by independently and randomly distributed sources and the action exerted on the point $\mathbf{x}_0$ by a source situated at the point $\mathbf{x}$ is defined by the function $S(\mathbf{x}_0, \mathbf{x})$, then the characteristic function $\Psi(u_1, u_2, l)$ of the distribution $\varphi(y, z, l)$ is determined by the following expression analogous to that given in [2 – 4]:

$$\ln \Psi(u_1, u_2, l) = -\rho \int \{1 - \exp[iu_1 S(0, \mathbf{x}) + iu_2 S(l, \mathbf{x})]\}\, d\mathbf{x} \tag{2}$$

where ρ is the mean source density, o and l are points on the observer line (along which we direct the x_1-axis) and the integration is performed over the space $\mathbf{x}$. The dimensionality of $\mathbf{x}$ depends on the type of sources. Writing $\varphi(y, z, l)$ in terms of the characteristic function (2) and passing to the limit in (1) we obtain, after the integration.

$$\mu^+(\tau) = \frac{1}{2}\rho \int S_1(0, \mathbf{x})\, \varphi[\tau - S(0, \mathbf{x})]\, d\mathbf{x} \tag{3}$$

$$S_1(x_0, \mathbf{x}) = \partial S(x_0, \mathbf{x})/\partial x_0$$

where $\varphi(y)$ is the density of distribution of probabilities of σ in an arbitrary point, and the integration is performed over the region of $\mathbf{x}$ for which $S_1(0, \mathbf{x}) > 0$. The mean number of downcrossings is $\mu^-(\tau) = \mu^+(\tau)$. Unlike the formulas given in [1, 4], formula (3) defines the mean number of crossings in terms of the one-dimensional distribution of values and the action of the source. Applying (3) to the trajectory of the derivative $\sigma_1(x_0) = \partial\sigma(x_0)/\partial x_0$ we obtain the following expression:

$$[\langle\lambda^+(\varkappa)\rangle]^{-1} = [\langle\lambda^-(\varkappa)\rangle]^{-1}_{\iota\iota} = \frac{1}{2}\rho \int S_2(0, \mathbf{x})\, \varphi_1[\varkappa - S_1(0, \mathbf{x})]\, d\mathbf{x} \tag{4}$$

$$S_2(x_0, \mathbf{x}) = \partial S_1(x_0, \mathbf{x})/\partial x_0$$

where $\varphi_1(t)$ denotes the distribution density of the probabilities of σ_1 at any point, and the integration is performed over the region of $\mathbf{x}$ for which $S_2(0, \mathbf{x}) > 0$

For the ergodic random fields the ratio of the sum of the lengths of the trajectory segments $\Sigma\,\xi(\tau)$ on which $\sigma \geqslant \tau$ to the total length L, is connected [1] with the one-dimensional distribution function by the expression

$$1 - \int_{-\infty}^{\tau} \varphi(y)\, dy = \lim_{L\to+\infty} \frac{1}{L} \sum \xi_i(\tau) = \mu^+(\tau)\, \langle\xi(\tau)\rangle \tag{5}$$

Relations (5) and (3) together define $\langle\xi(\tau)\rangle$.

We illustrate the application of the formulas (3) – (5) with a practical example, in which we determine the characteristics of the trajectories of the components σ_{12} and σ_{23} of an internal stress field σ generated by parallel independently distributed dislocations. The distributions $\varphi(y)$ and $\varphi_1(t)$ for this field are given in [3, 5]. The x_3-axis is directed along the line of dislocations, $S = S(x_0, x_1, x_2)$ [6] where x_0 is counted along the observer line and ρ denotes the mean dislocation density in the (x_1, x_2)-plane. When the mean number of crossings is being determined, the integral (3) can, in this case, only be obtained approximately.

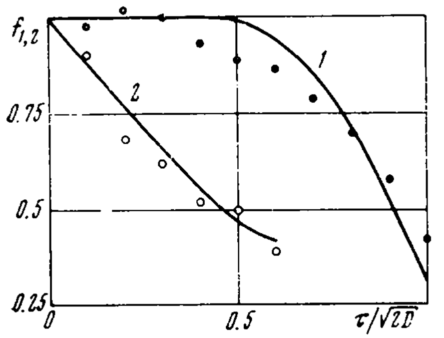

Fig. 2

Figure 2 depicts the following relations

$$1 - \mu^+\left(\frac{\tau}{\sqrt{2D}}\right)[\mu^+(0)]^{-1} = f_1\left(\frac{\tau}{\sqrt{2D}}\right), \quad \mu^+(0) = 0.29\,\rho^{1/2}$$

$$2 - \left\langle \xi\left(\frac{\tau}{\sqrt{2D}}\right)\right\rangle [\langle\xi(0)\rangle]^{-1} = f_2\left(\frac{\tau}{\sqrt{2D}}\right), \quad \langle\xi(0)\rangle = 1{,}7\rho^{-1/2}$$

for the component σ_{23} of a field generated by randomly distributed helical dislocations, where $D = \langle[\sigma_{23}]^2\rangle$.

The correctness of the above computations was checked by constructing a model of the field σ of 1000 screw dislocations parallel to the x_3-axis and situated on a 1×1 area on the $(x_1\, x_2)$-plane. Figure 2 shows the comparison of the relations $f_1(\tau/\sqrt{2D})$ and $f_2(\tau/\sqrt{2D})$ computed using the formulas (3) and (5) (white circles) with those obtained from the model (black dots). The modelling data give $\mu^+(0) = 0.28\,\rho^{1/2}$ and $\langle\xi(0)\rangle = 1.9\,\rho^{-1/2}$.

The following expression is obtained for $\langle\lambda^+(\varkappa)\rangle$

$$[\langle\lambda^+(\varkappa)\rangle]^{-1} = \frac{\alpha\rho\sqrt{A}}{(c^2+\varkappa^2)^{1/2}}\{[(c^2+\varkappa^2)^{1/2}+\varkappa]^{1/2} + [(c^2+\varkappa^2)^{1/2}-\varkappa]^{1/2}\} \tag{6}$$

where $A = G\,b/(2\pi\,k)$, G is the shear modulus of elasticity, b is the Burgers dislocation vector; $k = 1$ and $\alpha = 0.6$ for the helical dislocations (the σ_{23} component); $k = 1-\nu$ and $\alpha = 1.05$ for the edge dislocations (the σ_{12} component); ν is the Poisson's ratio and $c = \pi\,A\,\rho$.

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